Zadatak: Opišite riječima najmanje tri jezika te za njih konstruirajte regularne izraze.

Rješenje:
Prvi jezik:
Riječ jezika L nad abecedom $\Sigma = \{a, b, c\}$ sadrži točno dva znaka c,nakon svakog c slijedi neparan broj a-ova,nakon svakog b slijedi paran broj a-ova.
Regularni izraz:

$$a^*(b(aa)^+)^*ca(aa)^*(b(aa)^+)^*ca(aa)^*(b(aa)^+)^*$$

Drugi jezik:
Riječ jezika L nad abecedom $\Sigma = \{a, b, c\}$ ne sadrži izraz aba.
Regularni izraz:

$$(a + c + cb + bc + bb^+)^*$$

Treći jezik:
Riječ jezika L nad abecedom $\Sigma = \{a, b, c\}$ počima s parnim brojem a-ova,završava s neparnim brojem b-ova.
Regularni izraz:

$$(aa)^+(b(a + b + c)^*a + b(a + b + c)^*c + c(a + b + c)^*a + c(a + b + c)^*c)^*b(bb)^*$$

Zadatak:
a)Pojednostavni regularni izraz:

$$a^*(a^+ + \varepsilon)ab(b^+ + \varepsilon)^*$$

b)Dokaži ili opovrgni:

$$a^+(a^* + b^+)^+b^+ = a(a + b)^*b$$

c)Dokaži ili opovrgni:

$$a^+(a^* + b^+)^*b^+ = a(a + b)^+b$$

Rješenje:
a)
$a^*(a^+ + \varepsilon)ab(b^+ + \varepsilon)^* = a^*a^*ab(b^*)^* = a^*abb^* = a^+b^+$

b)

$\supseteq$

$a \subseteq a^* \subseteq (a^* + b^+)$
$b \subseteq b^+ \subseteq (a^* + b^+)$

$(a + b) \subseteq (a^* + b^+) \Rightarrow (a + b)^* \subseteq (a^* + b^+)^*$

$a \subseteq a^+ \subseteq a^+(a^* + b^+)$
$b \subseteq b^+$

$a(a + b)^*b \subseteq a^+(a^* + b^+)(a^* + b^+)^*b^+ = a^+(a^* + b^+)^+b^+$

$\subseteq$

$a \subseteq (a + b)$
$a^* \subseteq (a + b)^*$
$aa^* \subseteq a(a + b)^* \Rightarrow a^+ \subseteq a(a + b)^*$

$b \subseteq (a + b)$
$b^* \subseteq (a + b)^*$
$b^*b \subseteq (a + b)^*b \Rightarrow b^+ \subseteq (a + b)^*b$

$a \subseteq (a + b)$
$a^* \subseteq (a + b)^*$
$b \subseteq (a + b)$
$b^+ \subseteq (a + b)^+ \subseteq (a + b)^*$
$(a^* + b^+) \subseteq (a + b)^* \Rightarrow (a^* + b^+)^* \subseteq ((a + b)^*)^* = (a + b)^*$

$(a^* + b^+)(a^* + b^+)^* \subseteq (a + b)^*(a + b)^*$
$(a^* + b^+)^+ \subseteq (a + b)^*$

$a^+(a^* + b^+)^+b^+ \subseteq a(a + b)^*(a + b)^*(a + b)^*b \subseteq a(a + b)^*b$

c)
$L = a^+(a^* + b^+)^*b^+$
$G = a(a + b)^+b$

Jednakost ne vrijedi.
Kontraprimjer:

$ab \in L$
$ab \notin G$

Zadatak:

Konstruirajte minimalni DKA koji prepoznaje jezik opisan regularnim izrazom

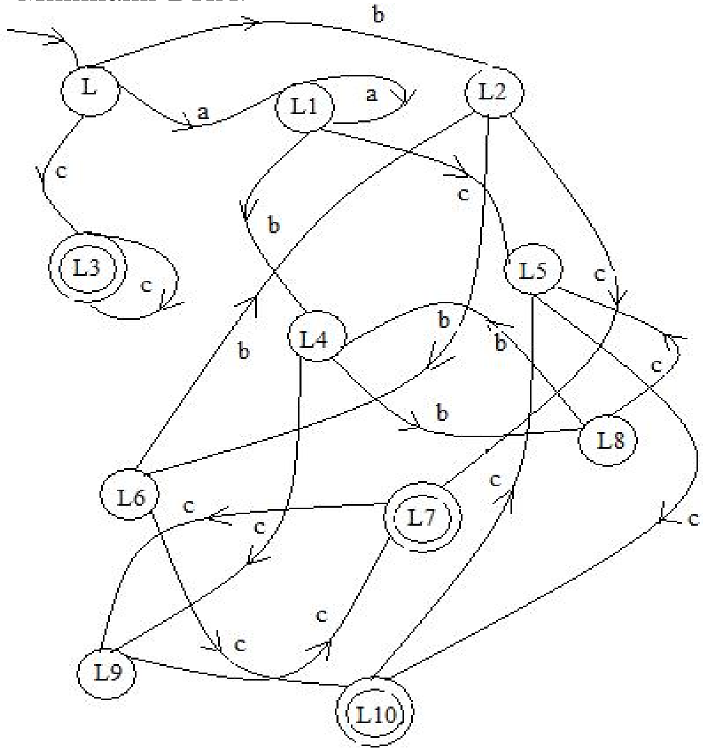
$$a^+b^*(cc)^+ + b^+c(cc)^* + a^*(bb)^*c^+$$

Rješenje:

$L = a^+b^*(cc)^+ + b^+c(cc)^* + a^*(bb)^*c^+$
$a^{-1}L = a^*b^*(cc)^+ + a^*(bb)^*c^+ = L_1$
$b^{-1}L = b^*c(cc)^* + b(bb)^*c^+ = L_2$
$c^{-1}L = c^* = L_3$
$a^{-1}L_1 = a^*b^*(cc)^+ + a^*(bb)^*c^+ = L_1$
$b^{-1}L_1 = b^*(cc)^+ + b(bb)^*c^+ = L_4$
$c^{-1}L_1 = c(cc)^* + c^* = L_5$
$a^{-1}L_2 = \oslash$
$b^{-1}L_2 = b^*c(cc)^* + (bb)^*c^+ = L_6$
$c^{-1}L_2 = (cc)^* = L_7$
$a^{-1}L_3 = \oslash$
$b^{-1}L_3 = \oslash$
$c^{-1}L_3 = c^* = L_3$
$a^{-1}L_4 = \oslash$
$b^{-1}L_4 = b^*(cc)^+ + (bb)^*c^+ = L_8$
$c^{-1}L_4 = c(cc)^* = L_9$
$a^{-1}L_5 = \oslash$
$b^{-1}L_5 = \oslash$
$c^{-1}L_5 = (cc)^* + c^* = L_{10}$
$a^{-1}L_6 = \oslash$
$b^{-1}L_6 = b^*c(cc)^* + b(bb)^*c^+ = L_2$
$c^{-1}L_6 = (cc)^* + c^* = L_{10}$
$a^{-1}L_7 = \oslash$
$b^{-1}L_7 = \oslash$
$c^{-1}L_7 = c(cc)^* = L_9$
$a^{-1}L_8 = \oslash$
$b^{-1}L_8 = b^*(cc)^+ + b(bb)^*c^+ = L_4$
$c^{-1}L_8 = c(cc)^* + c^* = L_5$
$a^{-1}L_9 = \oslash$
$b^{-1}L_9 = \oslash$
$c^{-1}L_9 = (cc)^* = L_7$
$a^{-1}L_{10} = \oslash$
$b^{-1}L_{10} = \oslash$

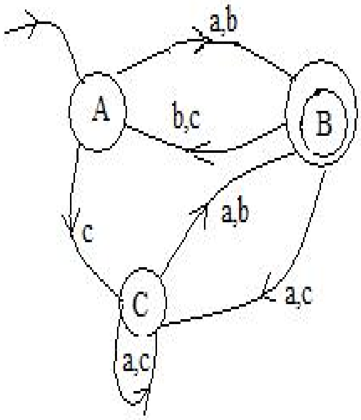
$c^{-1}L_{10} = c(cc)^* + c^* = L_5$
Praznu riječ prepoznaju jezici L_3,L_7,L_{10} i to će biti završna stanja automata.

Minimalni DKA:



Zadatak:
Dani NDKA prebacite u ekvivalentni DKA te konstruirajte skupovne jednadžbe.

Rješenje:
NDKA:

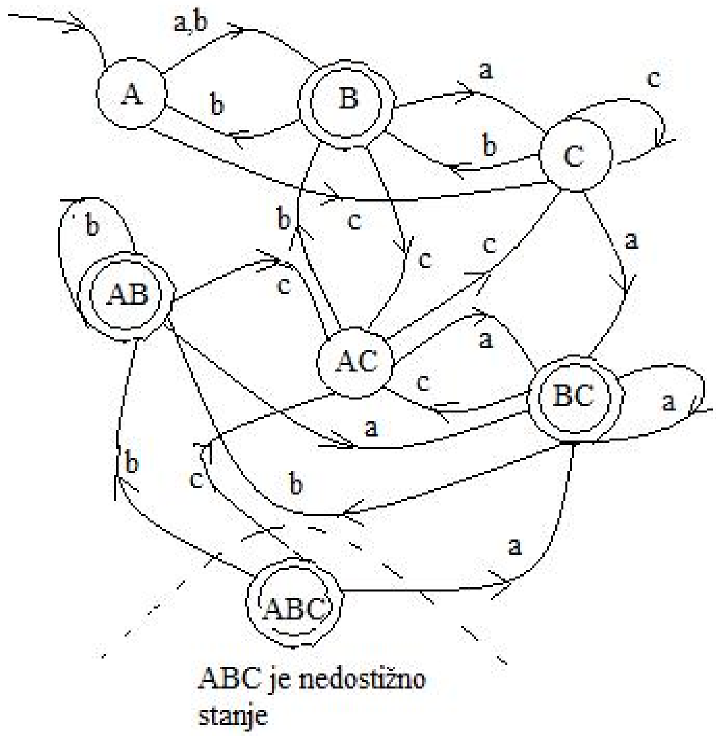


NDKA=($Q = \{A, B, C\}, \Sigma = \{a, b, c\}, \delta, Q_0 = \{A\}, F = \{B\}$)
DKA=($Q = 2^{\{A,B,C\}}, \Sigma = \{a, b, c\}, \delta', Q_0 = \{A\}, F = \{B, AB, BC, ABC\}$)

δ'....

0	A	B	C	AB	AC	BC	ABC
a	B	C	BC	BC	BC	BC	BC
b	B	A	B	AB	B	AB	AB
c	C	AC	C	AC	C	AC	AC

DKA:



Skupovne jednađbe:

$$A = aB + bB + cC$$
$$B = aC + bA + cAC + \varepsilon$$
$$C = aBC + bB + cC$$
$$AB = aBC + bAB + cAC + \varepsilon$$
$$AC = aBC + bB + cC$$
$$BC = aBC + bAB + cAC + \varepsilon$$
$$ABC = aBC + bAB + cAC + \varepsilon$$